**Fences and Functions –*Complete in Spiral Notebook***

Claire decided to plant a rectangular garden in her back yard using 30 pieces of fencing that were given to her by a friend. Each piece of fencing was a vinyl panel 1 yard wide and 6 feet high. Claire wanted to determine the possible dimensions of her garden, assuming that she would use all of the fencing and did not cut any of the panels. She began by placing ten panels (10 yards) parallel to the back side of her house and then calculated that the other dimension of her garden then would be 5 yards, as shown in the diagram below.

1. Claire looked at the 10 fencing panels lying on the ground and decided that she wanted to consider other possibilities for the dimensions of the garden. In order to organize her thoughts, she let *x* be the garden dimension parallel to the back of her house, measured in yards, and let *y* be the other dimension, perpendicular to the back of the house, measured in yards. She recorded the first possibility for the dimensions of the garden as follows: When *x* = 10, *y* = 5.



a. Explain why *y* must be 5 when *x* is 10.

b. Make a table showing the possibilities for *x* and *y*.

c. Find the perimeter of each of the possible gardens you listed in part b. What do you notice? Explain why this happens.

d. Did you consider *x* = 15 in part b? If *x* = 15, what must y be? What would Claire do with the fencing if she chose *x* = 15?

e. Can *x* be 16? What is the maximum possible value for *x*? Explain.

f. Write a formula relating the *y*-dimension of the garden to the *x*-dimension.

g. Make a graph of the possible dimensions of Claire’s garden.

h. What would it mean to connect the dots on your graph? Does connecting the dots make sense for this context? Explain.

i. As the *x*-dimension of the garden increases by 1 yard, what happens to the *y-*dimension? Does it matter what *x*-value you start with? How do you see this in the graph? In the table? In your formula? What makes the dimensions change together in this way?

2. After listing the possible rectangular dimensions of the garden, Claire realized that she needed to pay attention to the area of the garden, because area determines how many plants can be grown.

a. Does the area of the garden change as the *x*-dimension changes? Make a prediction, and explain your thinking.

b. Use grid paper to make accurate sketches for at least three possible gardens. How is the area of each garden represented on the grid paper?

c. Make a table listing all the possible *x*-dimensions for the garden and the corresponding areas. (To facilitate your calculations, you might want to include the *y*-dimensions in your table or add an area column to your previous table.)

d. Make a graph showing the relationship between the *x*-dimension and the area of the garden. Should you connect the dots? Explain.

e. Write a formula showing how to compute the area of the garden, given its *x*-dimension.

3. Because the area of Claire’s garden depends upon the *x*-dimension, we can say that the area is a function of the *x*-dimension. Let’s use *G* for the name of the function that uses each *x-*dimension an input value and gives the resulting garden area as the corresponding output value.

a. Use function notation to write the formula for the garden area function. What does *G*(11) mean? What is the value of *G*(11)? What line of your table, from #2, part c, and what point on your graph, from #2, part d, illustrates this same information?

b. The set of all possible input values for a function is called the ***domain*** of the function. What is the domain of the garden area function *G*? How is the question about domain related to the question about connecting the dots on the graph you drew for #2, part d?

c. The set of all possible output values is called the range of the function. What is the range of the garden area function G? How can you see the range in your table? In your graph?

d. As the x-dimension of the garden increases by 1 yard, what happens to the garden area? Does it matter what x-dimension you start with? How do you see this in the graph? In the table? Explain what you notice.

e. What is the maximum value for the garden area, and what are the dimensions when the garden has this area? How do you see this in your table? In your graph?

f. What is the minimum value for the garden area, and what are the dimensions when the garden has this area? How do you see this in your table? In your graph?

g. In deciding how to lay out her garden, Claire made a table and graph similar to those you have made in this investigation. Her neighbor Javier noticed that her graph had symmetry. Your graph should also have symmetry. Describe this symmetry by indicating the line of symmetry. What about the context of the garden situation causes this symmetry?

h. After making her table and graph, Claire made a decision, put up the fence, and planted her garden. If it had been your garden, what dimensions would you have used and why?

4. Later that summer, Claire’s sister-in-law Kenya mentioned that she wanted to use 30 yards of chain-link fence to build a rectangular pen for her two pet potbellied pigs. Claire experienced déjà vu and shared how she had analyzed how to fence her garden. As Claire explained her analysis, Kenya realized that her fencing problem was very similar to Claire’s.

a. Does the formula that relates the y-dimension of Claire’s garden to the x-dimension of her garden apply to the pen for Kenya’s pet pigs? Why or why not?

b. Does the formula that relates the area of Claire’s garden to its x-dimension apply to the pen for Kenya’s pet pigs? Why or why not?

c. Write a formula showing how to compute the area of the pen for Kenya’s pet pigs given the x-dimension for the pen.

d. Does the x-dimension of the pen for Kenya’s pet pigs have to be a whole number? Explain.

e. Let P be the function which uses the x-dimension of the pen for Kenya’s pet pigs as input and gives the area of the pen as output. Write a formula for P(x).

f. Make a table of input and output values for the function P. Include some values of the x-dimension for the pen that that could not be used as the x-dimension for Claire’s garden.

g. Make a graph of the function P.

h. Does your graph show any points with x-value less than 1? Could Kenya have made a pen with ½ yard as the x-dimension? If so, what would the other dimension be? How about a pen with an x-dimension 0.1 of a yard? How big is a potbellied pig? Would a potbellied pig fit into either of these pens?

i. Is it mathematically possible to have a rectangle with x-dimension equal to ½? How about a rectangle with x-dimension 0.1?

j. Of course, Kenya would not build a pen for her pigs that did not give enough room for the pigs to turn around or pass by each other. However, in analyzing the function *P* to decide how to build the pen, Kenya found it useful to consider all the input values that could be the *x*-dimension of a rectangle. She knew that it didn’t make sense to consider a negative number as the *x*-dimension for the pen for her pigs, but she asked herself if she could interpret an *x*-dimension of 0 in any meaningful way. She thought about the formula relating the *y*-dimension to the *x*-dimension and decided to include *x* = 0. What layout of fencing would correspond to the value *x* = 0? What area would be included inside the fence? Why could the shape created by this fencing layout be called a “degenerate rectangle”?

k. The value *x* = 0 is called a ***limiting case***. Is there any other limiting case to consider in thinking about values for the *x*-dimension of the pen for Kenya’s pigs? Explain.

l. Return to your graph of the function *P*. Adjust your graph to include all the values that could mathematically be the *x*-dimension of the rectangular pen even though some of these are not reasonable for fencing an area for potbellied pigs.

(Note: You can plot points corresponding to any limiting cases for the function using a small circle. To *include* the limiting case, *fill in the circle* to make a solid dot •. To *not include* the limiting case, but just use it to show that the graph does not continue beyond that point, *leave the circle open* O.)

5. Use your table, graph, and formula for the function *P* to answer questions about the pen for Kenya’s pet pigs.

a. Estimate the area of a pen of with an *x*-dimension of 10 feet (not yards). Explain your reasoning.

b. Estimate the *x*-dimension of a rectangle with an area of 30 square yards. Explain your reasoning.

c. What is the domain of the function *P*? How do you see the domain in the graph?

d. What point on the graph corresponds to the pen with maximum area?

e. What is the maximum area that the pen for Kenya’s pigs could have? Explain. What do you notice about the shape of the pen?

f. What is the range of the function *P*? How can you see the range in the graph?

g. How should you move your finger along the *x*-axis (right-to-left or left-to-right) so that the *x*-value increases as your move your finger? If you move your finger along the graph of the function *P* in this same direction, do the *x*-values of the points also increase? As the *x*-dimension of the pen for Kenya’s pigs increases, sometimes the area increases and sometimes the area decreases. Using your graph, determine the *x*-values such that the area increase as *x* increases. For what *x*-values does the area decrease as *x* increases?

When using tables and formulas, we often look at a function by examining only one or two points at a time, but in high school mathematics, it is important to begin to think about “the whole function,” that is, all of the input-output pairs. We’ve started working on this idea already by using a single letter such as *f*, *G*, or *P* to refer to the whole collection of input-output pairs. We’ll go further as we proceed with this investigation. We say that **two functions are equal** (as whole functions) if they have exactly the same input-output pairs. In other words, two functions are equal if they have the *same domain* **and** the *output values are the same for each input value in the domain*. From a graphical perspective, two functions are equal if their graphs have exactly the same points. Note that the graph of a function consists of all the points which correspond to input-output pairs, but when we draw a graph we often can show only some of the points and indicate the rest. For example, if the graph of the function is a line, we show part of the line and use arrowheads to indicate that the line continues without end.

6. The possibilities for the pen for Kenya’s pet pigs and for Claire’s garden are very similar in some respects but different in others. These two situations involve different *functions*, even though the *formulas* are the same.

a. If Kenya makes the pen with maximum area, how much more area will the pen for her pet pigs have than Claire’s garden of maximum area? How much area is that in square feet?

b. What could Claire have done to build her garden with the same area as the maximum area for Kenya’s pen? Do you think this would have been worthwhile?

c. Consider the situations that led to the functions *G* and *P* and review your tables, graphs, formulas related to the two functions. Describe the similarities and differences between Kenya’s pig pen problem and Claire’s garden problem. Your response should include a discussion of *domain* and *range* for the two functions.