

Literacy

APS

UNit of Study-6

**gSE Algebra I**

**Mathematics**

**Teacher Resource Guide**

**Describing Data:**

***10 instructional days***

***(March 20 – April 20 ) A/B Schedule***

***(November 28 – December 11) 4X4 Fall Schedule***

***7 instructional days***

***(April 9 – April 17) 4X4 Spring Schedule***

In this unit students will summarize, represent, and interpret data on a single count or measurement variable. Students will also summarize, represent, and interpret data on two categorical and quantitative variables. Students will interpret linear models.

The goal of this unit is to help students develop a deep understanding of how to represent quantitative and categorical data in ways that help us make sense of it, and enable us to make comparisons. Students will also learn how to create models that represent quantitative data, and interpret those models to determine what they are really showing. It is important that teachers help students make the connections between linear functions and the models used to represent quantitative data.

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| **Standards Addressed** | **Duration: maximum of 12 instructional days** |
| **Georgia Standards of Excellence – Mathematics**  **Content Standards**  (The following icons indicate cluster emphasis. Please note that 70% of the time should be focused on the Major Content. ◊ Major Content □ Supporting Content)  ◊***Summarize, represent, and interpret data on a single count or measurement variable.***  MGSE9-12.S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).  MGSE9-12.S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, mean absolute deviation, ~~standard deviation~~) of two or more different data sets.  MGSE9-12.S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).  ◊***Summarize, represent, and interpret data on two categorical and quantitative variables.***  MGSE9-12.S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.  MGSE9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.  MGSE9-12.S.ID.6a Decide which type of function is most appropriate by observing graphed data, charted data, or by analysis of context to generate a viable (rough) function of best fit. Use this function to solve problems in context. Emphasize linear, quadratic and exponential models. MGSE9-12.S.ID.6c Using given or collected bivariate data, fit a linear function for a scatter plot that suggests a linear association.  ◊***Interpret linear models.***  MGSE9-12.S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.  MGSE9-12.S.ID.8 Compute (using technology) and interpret the correlation coefficient “r” of a linear fit. (For instance, by looking at a scatterplot, students should be able to tell if the correlation coefficient is positive or negative and give a reasonable estimate of the “r” value.) After calculating the line of best fit using technology, students should be able to describe how strong the goodness of fit of the regression is, using “r”.  MGSE9-12.S.ID.9 Distinguish between correlation and causation.  **Standards for Mathematical Practice**  ***SMP 1. Make sense of problems and persevere in solving them***.  Students use problem-solving approaches to recognize and further explore quadratic functions. They make connections and synthesize characteristics of quadratic functions within context.   |  |  | | --- | --- | | **Students:** | **Because Teachers:** | | * Read the task carefully. * Draw pictures, diagrams, tables, or use objects to make sense of the task. * Discuss the meaning of the task with classmates. * Make choices about which solution path to take. * Try out potential solution paths and make changes as needed. * Check answers and makes sure solutions are reasonable and make sense. * Explore other ways to solve the task. * Persist in efforts to solve challenging tasks, even after reaching a point of frustration. | * Provide rich tasks aligned to the standards. * Allow students time to initiate a plan; uses question prompts as needed to assist students in developing a pathway. * Continually ask students if their plans and solutions make sense. * Question students to see connections to previous solution attempts and/or tasks to make sense of current task. * Consistently ask students to defend and justify their solution by comparing solution paths. * Provide appropriate time for students to engage in the productive struggle of problem-solving. * Differentiate to keep advanced students challenged during work time. |   ***SMP2: Reason abstractly and quantitatively.***  Students use quadratic equations and functions to solve real life problems. Students recognize real life context that can be represented with a quadratic equation (decontextualize), decide on the most appropriate method for solving that equation, and then interpret that solution back in the context of the problem (contextualize).   |  |  | | --- | --- | | **Students:** | **Because Teachers:** | | * Use mathematical symbols to represent situations * Take quantities out of context to work with them (decontextualizing) * Put quantities back in context to see if they make sense (contextualizing) * Consider units when determining if the answer makes sense in terms of the situation | * Provide a variety of problems in different contexts that allow students to arrive at a solution in different ways * Use think aloud strategies as they model problem solving * Attentively listen for strategies students are using to solve problems |   ***SMP 3. Construct viable arguments and critique the reasoning of others.***  Students construct and analyze arguments regarding the properties of quadratic functions. They make conjectures about the meaning of solutions and investigate their veracity based on the context of problems.   |  |  | | --- | --- | | **Students:** | **Because Teachers:** | | * Make and tests conjectures. * Explain and justifies their thinking using words, objects, and drawings. * Listen to the ideas of others and decides if they make sense. * Ask useful questions. * Identify flaws in logic when responding to the arguments of others. * Elaborate with a second sentence (spontaneously or prompted by the teacher or another student) to explain their thinking and connect it to their first sentence. * Talks about and asks questions about each other’s thinking, in order to clarify or improve their own mathematical understanding. * Revise their work based upon the justification and explanations of others. | * Encourage students to use proven mathematical understandings, (definitions, properties, conventions, theorems, etc.) to support their reasoning. * Question students so they can tell the difference between assumptions and logical conjectures. * Ask questions that require students to justify their solution and their solution pathway. * Prompt students to respectfully evaluate peer arguments when solutions are shared. * Ask students to compare and contrast various solution methods. * Create various instructional opportunities for students to engage in mathematical discussions (whole group, small group, partners, etc.). |   ***SMP4: Model with Mathematics***  Students formulate mathematical models that demonstrate the functional relationships that arise in real world context. They represent and analyze relationships using tables, diagrams and equations and describe graphical models. They use function notation to articulate the properties of functions.   |  |  | | --- | --- | | **Students:** | **Because Teachers:** | | * Use mathematical models (i.e. formulas, equations, symbols) to solve problems in the world * Use appropriate tools such as objects, drawings, and tables to create mathematical models * Make connections between different mathematical representations (concrete, verbal, algebraic, numerical, graphical, pictorial, etc.) * Check to see if an answer makes sense within the context of a situation and changing the model as needed | * Provide opportunities for students to solve problems in real life contexts * Identify problem solving contexts connected to student interests |   ***SMP5: Use Appropriate Tools Strategically***  Students create and use tables. They use technology to graph and analyze quadratic relationships. Students use the coordinate plane to help them understand what the solution(s) means in terms of the context of the problem.   |  |  | | --- | --- | | **Students:** | **Because Teachers:** | | * Use technological tools to explore and deepen understanding of concepts * Decide which tool will best help solve the problem. Examples may include:   + Calculator   + Concrete models   + Digital Technology   + Pencil/paper   + Ruler, compass, protractor * Estimate solutions before using a tool * Compare estimates to solutions to see if the tool was effective | * Make a variety of tools readily accessible to students and allowing them to select appropriate tools for themselves * Help students understand the benefits and limitations of a variety of math tools |   ***SMP6: Attend to precision.***  Students work on algebraic and graphical representation of solutions. They pay careful attention to precision especially in labeling axes and clarifying the correspondence with quantities in a problem. They understand that obtaining accurate solutions is key in ensuring that algebraic and graphical solutions agree. Students provide carefully articulated explanations using academic vocabulary involving quadratic equations and inequalities.   |  |  | | --- | --- | | **Students:** | **Because Teachers:** | | * Communicate precisely using clear language and accurate mathematics vocabulary * Decide when to estimate or give an exact answer * Calculate accurately and efficiently, expressing answers with an appropriate degree of precision * Use appropriate units; appropriately labeling diagrams and graphs | * Explicitly teach mathematics vocabulary * Insist on accurate use of academic language from students * Model precise communication * Require students to answer problems with complete sentences, including units * Provide opportunities for students to check the accuracy of their work |   ***SMP 7. Look for and make use of structure.***  Students seek patterns or structures to model equivalent expressions and. They identify patterns and see relationships between products and factors of quadratic expressions and equations. They generate equivalent expressions and solve equations by factoring, completing the square, and the quadratic formula.   |  |  | | --- | --- | | **Students:** | **Because Teachers:** | | * Find structure and patterns in numbers * Find structure and patterns in diagrams and graphs * Use patterns to make rules about math * Use these math rules to help them solve problems | * Provide sense making experiences for all students * Allow students to do the work of using structure to find the patterns for themselves rather than doing this work for students |   **Note:** All of the Standards for Mathematical Practice (SMPs) are critical to students fully and appropriately attending to the content. Not all SMPs will occur in every lesson, however SMPs 1, 3, and 6 should be regularly apparent. ***All SMPs should be taught in tandem with the content standards.*** |
| **Enduring Understandings** | In order to support deep conceptual learning it is important that student leave this unit experience with the following understandings:   * Linear and exponential equations in one variable can be used in a contextual situation to solve problems. * Real world situations can be represented symbolically and graphically. * Arithmetic and Geometric sequences are functions. * Functions can be used to model relationships between two quantities. |
| **Social Emotional Learning** | **Second Step Skill and**  **Gradual Release of Responsibility Lesson Alignment**  SEL skills taught in the Second Step curriculum can be reinforced throughout the instructional day throughout lessons, at transitions, and when issues arise utilizing the ARR strategy above. The following is an alignment between Gradual Release and Second Step components.   |  |  |  |  | | --- | --- | --- | --- | | **“I DO”**  **Focused Lesson**  (Self-Management) | **“WE DO”**  **Guided Practice**  (Self-Awareness & Social Awareness) | **“YOU DO”**  **Collaborative Practice**  (Relationship Skills & Social Awareness) | **“YOU DO”**  **Independent Practice**  (Responsible Decision Making) | | * Follow Listening Rules * Use Self-Talk to stay focused * Put on * Attent-o-scopes | * Being Assertive to ask questions * Use Self-Talk to stay focused * Use Active Listening Strategies | * Use Active Listening and Conversation Skills * Use empathy to look for clues about classmates’ perspectives and feelings * Compassion | * Use Self-Talk to stay focused and persevere on a task * Confidence * Calm Down Strategy | |

# **Lesson One Progression**

**Duration 2-3 days**

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| **Focus Standard(s)** | | | |
| ◊***Summarize, represent, and interpret data on a single count or measurement variable.***  **MGSE9-12.S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).  **MGSE9-12.S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, mean absolute deviation, ~~standard deviation~~) of two or more different data sets.  **MGSE9-12.S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | | | |
| **Performance Objectives** | | | |
| **As a result of their engagement with this unit…** | | | |
| **SWBAT** construct a dot plot, histogram, and box plot on the real number line **IOT** determine whether a particular representation preserves all the data values or present only the general characteristics of a data set.  **SWBAT** describe the center and spread of a data distribution **IOT** compare the distributions of two or more data sets  **SWBAT** Interpret the context of data sets and identify extreme data points **IOT** predict the effect outliers have and explain why distributions take on a particular shape. | | | |
| **Building Coherence** | | | |
| “The Standards are designed around coherent progressions from grade to grade. Learning is carefully connected across grades so that students can build new understanding onto foun    dations built in previous years. Each standard is not a new event, but an extension of previous learning.”  [***http://achievethecore.org/page/1088/coherence***](http://achievethecore.org/page/1088/coherence)  **Across Grades:**  **Within Grades:** | | | |
| **Terms and Definitions** | | | |
| * **Association**. A connection between data values. * **Bivariate data**. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team. * **Box Plot**. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data. * **Box-and-Whisker Plot**. A diagram that shows the five-number summary of a distribution. (Five-number summary includes the minimum, lower quartile (25th percentile), median (50th percentile), upper quartile (75th percentile), and the maximum. In a modified box plot, the presence of outliers can also be illustrated. * **Categorical Variables.** Categorical variables take on values that are names or labels. The color of a ball (e.g., red, green, blue), gender (male or female), year in school (freshmen, sophomore, junior, senior). These are data that cannot be averaged or represented by a scatter plot as they have no numerical meaning. * **Center.** Measures of center refer to the summary measures used to describe the most “typical” value in a set of data. The two most common measures of center are median and the mean. * **Data**. Values of qualitative or quantitative variables belonging to a set of items. * **Data set.** A collection of data. * **Dot plot.** A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. * Mean Absolute Deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20. * **Outlier.** Sometimes, distributions are characterized by extreme values that differ greatly from the other observations. These extreme values are called outliers. As a rule, an extreme value is considered to be an outlier if it is at least 1.5 interquartile ranges below the lower quartile (Q1), or at least 1.5 interquartile ranges above the upper quartile (Q3).   **OUTLIER if the values lie outside these specific ranges:**  Q1 – 1.5 • IQR  Q3 + 1.5 • IQR   * **Quantitative Variables**. Numerical variables that represent a measurable quantity. For example, when we speak of the population of a city, we are talking about the number of people in the city – a measurable attribute of the city. Therefore, population would be a quantitative variable. Other examples: scores on a set of tests, height and weight, temperature at the top of each hour. | | * **First Quartile (Q1).** The “middle value” in the lower half of the rank-ordered data * **Five-Number Summary**. Minimum, lower quartile, median, upper quartile, maximum. * **Frequency**. The number of occurrences of a repeating event per unit time. * **Histogram.** Graphical display that subdivides the data into class intervals and uses a rectangle to show the frequency of observations in those intervals—for example you might do intervals of 0-3, 4-7, 8-11, and 12-15 * **Interquartile Range**. A measure of variation in a set of numerical data. The interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. * **Second Quartile (Q2)**. The median value in the data set. * **Shape**. The shape of a distribution is described by symmetry, number of peaks, direction of skew, or uniformity. * **Symmetry**. A symmetric distribution can be divided at the center so that each half is a mirror image of the other. * **Number of Peaks**. Distributions can have few or many peaks. Distributions with one clear peak are called unimodal and distributions with two clear peaks are called bimodal. Unimodal distributions are sometimes called bell-shaped. * **Direction of Skew**. Some distributions have many more observations on one side of graph than the other. Distributions with a tail on the right toward the higher values are said to be skewed right; and distributions with a tail on the left toward the lower values are said to be skewed left. * **Uniformity**. When observations in a set of data are equally spread across the range of the distribution, the distribution is called uniform distribution. A uniform distribution has no clear peaks. * **Univariate data**. Data that involves only one variable, such as the daily temperature in degrees Fahrenheit or travel time in minutes. If you have two sets of data, such as ice cream sales vs. temperature, it is called bivariate data. * **Spread**. The spread of a distribution refers to the variability of the data. If the data cluster around a single central value, the spread is smaller. The further the observations fall from the center, the greater the spread or variability of the set. (range, interquartile range, Mean Absolute Deviation, and Standard Deviation measure the spread of data) * **Third quartile**. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. * **Trend**. A change (positive, negative or constant) in data values over time. | |
| **Guiding Question(s)** | | | |
| * How do I summarize, represent, and interpret data on a single count or measurement variable? * When making decisions or comparisons, what factors are important for me to consider in determining which statistics to compare, which graphical representation to use, and how to interpret the data? * How can I use visual representations and measures of center and spread to compare two data sets? * How do I summarize, represent, and interpret data on two categorical and quantitative variables? | | | |
| **Interpretations and Reminders** | | | |
| * Measures of center and spread for data sets without outliers are the mean and mean absolute deviation, whereas median and interquartile range are better measures for data sets with outliers. * When you compare two or more data sets, focus on four features:   + Center. Graphically, the center of a distribution is the point where about half of the observations are on either side.   + Spread. The spread of a distribution refers to the variability of the data. If the observations cover a wide range, the spread is larger. If the observations are clustered around a single value, the spread is smaller.      * + Shape. The shape of a distribution is described by symmetry, skewness, number of peaks, etc.      * + Unusual features. Unusual features refer to gaps (areas of the distribution where there are no observations) and outliers.      * Although this domain addresses both categorical and quantitative data, there is no reference in the Standards 1 - 4 to categorical data. Note that Standard 5 in the next cluster (Summarize, represent, and interpret data on two categorical and quantitative variables) addresses analysis for two categorical variables on the same subject. To prepare for interpreting two categorical variables in Standard 5, this would be a good place to discuss graphs for one categorical variable (bar graph, pie graph) and measure of center (mode). * Help students to clearly distinguish between categorical and numerical variables by providing multiple examples of each type. Remind students about the four step statistical process that should have been introduced in middle school. In high school, students should become proficient in generating meaningful questions. * **Four Step Statistical Process**   1) Formulate a question that can be answered by data  2) Design and implement a plan that collects appropriate data  3) Analyze the data by graphical and/or numerical methods  4) Interpret the analysis in the context of the original question   * Students should formulate meaningful questions in the first step of the four-step process. This takes time and lots of practice, and leads to real-world contexts. * Some students may need to begin with “well-behaved” data sets. As they progress in their understanding and work with data, begin to include data sets with outliers and non-Normal shapes. | | | |
| **Misconceptions** | | | |
| Students may ...   * Perceive histograms as two-dimensional graphs that must have two variables, and thus tend to interpret a histogram as a two-variable scatterplot. * Perceive histograms as displays of raw data on Y with each bar standing for an individual observation and individual case or time on X (case or time series plots). * Misunderstand the difference between the points plotted on a scatter graph (to represent a pair of variables) and the points on a line graph (where different values of a single variable are shown, often to show change over a time period). * Believe that a bar graph and a histogram are the same. A bar graph is appropriate when the horizontal axis has categories and the vertical axis is labeled by either frequency (e.g., book titles on the horizontal and number of students who like the respective books on the vertical) or measurement of some numerical variable (e.g., days of the week on the horizontal and median length of root growth of radish seeds on the vertical). A histogram has units of measurement of a numerical variable on the horizontal (e.g., ages with intervals of equal length). * Think that the lengths of the intervals of a boxplot (min,Q1), (Q1,Q2), (Q2,Q3), (Q3,max) are related to the number of subjects in each interval. Students should understand that each interval theoretically contains one-fourth of the total number of subjects. Sketching an accompanying histogram and constructing a live boxplot may help in alleviating this misconception. * Think that all bell-shaped curves are normal distributions. For a bell-shaped curve to be Normal there needs to be 68% of the distribution within 1 standard deviation of the mean, 95% within two, and 99.7% within 3 standard deviations. * Be unable to select which measure of central tendency is most appropriate for a given problem context. Students therefore cannot select the appropriate statistic for a given distribution of data. * Confuse measure of central tendency definitions. * Not understand the concept of data itself; they can calculate a mean but do not necessarily associate the numbers they add and divide with any type of real-world occurrence. * Struggle to grasp the idea that an average is a representation of the values contained in a data set. * See finding the average as the end goal of a task, not looking beyond the algorithm to thinking about what kinds of information this number represents.   [*http://www.katm.org/flipbooks/HS%20FlipBook%20Final%20CCSS%202014.pdf*](http://www.katm.org/flipbooks/HS%20FlipBook%20Final%20CCSS%202014.pdf) | | | |
| **Learning Progression (Suggested Learning Experiences)** | | | |
| **Procedural Fluency:** Fluency strategies are useful to activate student voice, solicit prior knowledge and develop fluency based on conceptual understandings. The fluency practice below is not intended to be the only experience students have within this unit. For additional fluency practice strategies see the table at the end of this document.    Using the following website, <https://www.ixl.com/math/algebra-1/interpret-bar-graphs-line-graphs-and-histograms> give students a daily practice problem where they have to interpret histograms Let the students write their answers on a small whiteboard.  ---------------------------------------------------------------------------------------------------------------------------------------------------  **Graduated Measure**  The “Graduated Measure” is an ***initial*** opportunity to diagnose students’ ***level of comfort*** with the progression’s content prior to the progression beginning. Students should be afforded the opportunity to select and attempt ONE of the three problems on the “Graduated Measure” sheet of their choosing. The data gathered from this formative assessment and other data should be used to guide instruction (tasks, manipulatives, etc.), address misconceptions, and plan for differentiation. It is not evaluative in nature. Please note that success on the Graduated Measure assessment does not indicate proficiency of the standards, and students will still need to engage with the unit’s content to determine proficiency.  ***Student Directions:*** Select a question/problem from the table below that you feel best equipped to answer successfully.   |  |  |  | | --- | --- | --- | | **Beginning** | **Developing** | **Proficient** | | Display each set of data in a stem-and-leaf plot.  {7, 2, 3, 11, 20, 21, **17,15,15,14}** | Which statistic can not be determined from a box plot representing the scores on a math test in Mrs. DeRidder’s algebra class? | Robin collected data on the number of hours she watched television on Sunday through Thursday nights for a period of 3 weeks. The data are shown in the table below.    Using an appropriate scale on the number line below, construct a box plot for the 15 values. |   **Gradual Release of Responsibility**    ◊***Summarize, represent, and interpret data on a single count or measurement variable.***  **MGSE9-12.S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).  **SWBAT** construct a dot plot, histogram, and box plot on the real number line **IOT** determine whether a particular representation preserves all the data values or present only the general characteristics of a data set.  **Focus Lesson**  Provide the following notes to students. Giving an example of each type of graphical display, they may encounter. Make sure you emphasize the vocabulary.          **Guided Practice**  Give the students the following problems. Work the problem with students in a whole group    **\*Teacher Note:** Engage students in deep conversations around the analysis of the equation, graph and table as it relates to the context  **Collaborative Practice**  Place students into groups of 2-3 to work the following tasks. Make sure the students focus on multiple representation and share their representations with other groups. Make sure during the sharing phase students explain and compare their answers ( *SMP 4,*  *SMP 6, SMP 7)*  **Independent Practice**        ◊***Summarize, represent, and interpret data on a single count or measurement variable.***  **MGSE9-12.S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, mean absolute deviation, ~~standard deviation~~) of two or more different data sets.  **SWBAT** describe the center and spread of a data distribution **IOT** compare the distributions of two or more data sets  **Focus Lesson**  Share the following steps with students. You can create a notes page as you explain the median, mean, interquartile range and mean absolute deviation.  Two measures of center are the mean and median.  **Finding the Mean**  1. Find the sum of the data values.  2. Divide the sum by the number of data points. This is the mean.  • The mean is useful when data sets do not contain values that vary greatly.  • Median is a second measure of center.  **Finding the Median**  1. First arrange the data from least to greatest.  2. Count the number of data points. If there is an even number of data points, the  median is the average of the two middle-most values. If there is an odd number  of data points, the median is the middle-most value.  Mean absolute deviation and interquartile range describe variability.  **Finding the Mean Absolute Deviation**  1. Find the mean.  2. Calculate the absolute value of the difference between each data value and the mean.  3. Determine the average of the differences found in step 2. This average is the mean absolute deviation.  • The mean absolute deviation takes the average distance of the data points from the mean.  • This summarizes the variability of the data using one number.  **Finding the Interquartile Range**  1. Arrange the data from least to greatest.  2. Count the number of data points in the set.  3. Find the median of the data set. The median divides the data into two halves: the lower half and the upper half.  4. Find the middle-most value of the lower half of the data. The data to the left represents the first quartile, Q1.  5. Find the middle-most value of the upper half of the data. The data to the right is the third quartile, Q 3.  6. Calculate the difference between the two quartiles, Q 3 – Q 1. The interquartile range is the difference between the third and first quartiles.  **Guided Practice**  Give the students the following problem. Work the problem with students in a whole group. Emphasize the process of finding the center and spread for each. Adapted from Walch    **\*Teacher Note:** Engage students in deep conversations around the analysis of the equation, graph and table as it relates to the context.    **Collaborative Practice**  Place students into groups of 2-3 to work the following tasks. Make sure the students focus on multiple representation and share their representations with other groups. Make sure during the sharing phase students explain and compare their answers ( *SMP 4,*  SMP 5, *SMP 6)*.    **Independent Practice**        ◊***Summarize, represent, and interpret data on a single count or measurement variable.***  **MGSE9-12.S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).  **SWBAT** Interpret the context of data sets and identify extreme data points **IOT** predict the effect outliers have and explain why distributions take on a particular shape.  **Focus Lesson**  Share with students the concept of shape. Project the following picture and discuss skewness.  Image result for shape of a graph statisticsEmphasize the use description of each skewness. Remind students about center and spread as well.  **Guided Practice**  Guide the students through the following process in a whole group setting. Make you engage students in deep conversations around the analysis of the statistics it relates to the context.  **\*Teacher Note:** Engage students in deep conversations around the analysis of the equation, graph and table as it relates to the context.    **Collaborative Practice**  Place students into groups of 2-3 to work the following tasks. Make sure the students focus on multiple representation and share their representations with other groups. Make sure during the sharing phase students explain and compare their answers ( *SMP 4,*  *SMP 6)*.    **Independent Practice**          The Progression Assessment is a 5-question mini assessment coving the standards within this progression only. Use it to guide the reengagement, differentiation, and enrichment opportunities provided to students. | | |
| **Differentiated Supports** | | | |
| *Learning Difficulty* | Jigsaw strategy: Students are divided into four groups to write, strategize, simplify and evaluate their own solutions to their equations.  * Allow students to use the TRACE feature on the graphing calculator to solve equations. Students will see the solution is the x value that corresponds to the same two y-values in the table. * Provide students a graphic organizer. * Create a “Modeling Contexts” Choice Board that allows students to attempt problems based on their comfort level. Include scenarios that can be modeled by various types of equations. | | |
| *High-Achieving* | **Extension:**  Have students calculate the standard deviation of a data set and compare the Mean Absolute Deviation with the Standard Deviation in terms of measuring the spread of a data set. | | |
| *English Language Learners* | * Have students write their understandings in a math journal. You may want to consider allowing EL students to write notes in their first language and annotate by identifying math specific words for which there are no direct translations. * Allow students to create pictorial flash cards for new math words. * Create a graphic organizer that outlines the characteristics of a quadratic function. | | |
| **Online/Print Resources** | | | |
| *Digital Resources* | Khan Academy – Mean Absolute Deviation <https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-data-statistics/cc-6-mad/v/mean-absolute-deviation> | | |
| *Print Resources* | Florida State University | | |
| *Manipulatives and Tools* | http://scene7.samsclub.com/is/image/samsclub/0003331719206_A?$img_size_380x380$  TI-84 Handheld Wabbitemu App  Image result for desmos exponential growth  <https://www.desmos.com/calculator> | | |
| **Textbook Alignment** | | | |
| *Algebra I,* McGraw-Hill Education | | MGSE9-12.S.ID.1 🡪 Lessons: 0-13, 12-3, 12-4  MGSE9-12.S.ID.2 🡪 Lessons: 12-2, 12-3, 12-4  Extend: 12-8  MGSE9-12.S.ID.3 🡪 Lessons: 12-3, 12-4 | |

# **Lesson Two Progression**

**Duration 3-4 days**

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| **Focus Standard(s)** | |
| ◊***Summarize, represent, and interpret data on two categorical and quantitative variables.***  **MGSE9-12.S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.  **MGSE9-12.S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.  **MGSE9-12.S.ID.6a** Decide which type of function is most appropriate by observing graphed data, charted data, or by analysis of context to generate a viable (rough) function of best fit. Use this function to solve problems in context. Emphasize linear, quadratic and exponential models.  **MGSE9-12.S.ID.6c** Using given or collected bivariate data fit a linear function for a scatter plot that suggests a linear association. | |
| **Performance Objectives** | |
| **As a result of their engagement with this unit…** | |
| * **SWBAT** summarize categorical data for two categories in two-way frequency tables **IOT i**nterpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies) and recognize possible associations and trends in the data. * **SWBAT** represent data on two quantities on a scatter plot **IOT** generate a rough function of best fit. | |
| **Building Coherence** | |
| “The Standards are designed around coherent progressions from grade to grade. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.”    [***http://achievethecore.org/page/1088/coherence***](http://achievethecore.org/page/1088/coherence)  **Across Grades:**  **Within Grades:** | |
| **Terms and Definitions** | |
| * **Association**. A connection between data values. * **Bivariate data**. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team. * **Box Plot**. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data. * **Box-and-Whisker Plot**. A diagram that shows the five-number summary of a distribution. (Five-number summary includes the minimum, lower quartile (25th percentile), median (50th percentile), upper quartile (75th percentile), and the maximum. In a modified box plot, the presence of outliers can also be illustrated. * **Categorical Variables.** Categorical variables take on values that are names or labels. The color of a ball (e.g., red, green, blue), gender (male or female), year in school (freshmen, sophomore, junior, senior). These are data that cannot be averaged or represented by a scatter plot as they have no numerical meaning. * **Center.** Measures of center refer to the summary measures used to describe the most “typical” value in a set of data. The two most common measures of center are median and the mean. * **Conditional Frequencies.** The relative frequencies in the body of a two-way frequency table. * **Correlation Coefficient.** A measure of the strength of the linear relationship between two variables that is defined in terms of the (sample) covariance of the variables divided by their (sample) standard deviations. * **Dot plot.** A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. * **Joint Frequencies.** Entries in the body of a two-way frequency table. * **Line of Best Fit (trend or regression line).** A straight line that best represents the data on a scatter plot. This line may pass through some of the points, none of the points, or all of the points. Remind students that an exponential model will produce a curved fit. * **Marginal Frequencies**. Entries in the "Total" row and "Total" column of a two-way frequency table. * Mean Absolute Deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20. * **Outlier.** Sometimes, distributions are characterized by extreme values that differ greatly from the other observations. These extreme values are called outliers. As a rule, an extreme value is considered to be an outlier if it is at least 1.5 interquartile ranges below the lower quartile (Q1), or at least 1.5 interquartile ranges above the upper quartile (Q3).   **OUTLIER if the values lie outside these specific ranges:**  Q1 – 1.5 • IQR  Q3 + 1.5 • IQR   * **Quantitative Variables**. Numerical variables that represent a measurable quantity. For example, when we speak of the population of a city, we are talking about the number of people in the city – a measurable attribute of the city. Therefore, population would be a quantitative variable. Other examples: scores on a set of tests, height and weight, temperature at the top of each hour. | * **First Quartile (Q1).** The “middle value” in the lower half of the rank-ordered data * **Five-Number Summary**. Minimum, lower quartile, median, upper quartile, maximum. * **Histogram.** Graphical display that subdivides the data into class intervals and uses a rectangle to show the frequency of observations in those intervals—for example you might do intervals of 0-3, 4-7, 8-11, and 12-15 * **Interquartile Range**. A measure of variation in a set of numerical data. The interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. * **Scatter plot.** A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot. If you are looking for values that fall within the range of values plotted on the scatter plot, you are interpolating. If you are looking for values that fall beyond the range of those values plotted on the scatter plot, you are extrapolating. * **Second Quartile (Q2)**. The median value in the data set. * **Shape**. The shape of a distribution is described by symmetry, number of peaks, direction of skew, or uniformity. * **Symmetry**. A symmetric distribution can be divided at the center so that each half is a mirror image of the other. * **Number of Peaks**. Distributions can have few or many peaks. Distributions with one clear peak are called unimodal and distributions with two clear peaks are called bimodal. Unimodal distributions are sometimes called bell-shaped. * **Direction of Skew**. Some distributions have many more observations on one side of graph than the other. Distributions with a tail on the right toward the higher values are said to be skewed right; and distributions with a tail on the left toward the lower values are said to be skewed left. * **Uniformity**. When observations in a set of data are equally spread across the range of the distribution, the distribution is called uniform distribution. A uniform distribution has no clear peaks. * **Spread**. The spread of a distribution refers to the variability of the data. If the data cluster around a single central value, the spread is smaller. The further the observations fall from the center, the greater the spread or variability of the set. (range, interquartile range, Mean Absolute Deviation, and Standard Deviation measure the spread of data) * **Third quartile**. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. * **Trend**. A change (positive, negative or constant) in data values over time. |
| **Guiding Question(s)** | |
| * How do I summarize, represent, and interpret data on two categorical and quantitative variables? * How can we use functions to model and interpret bivariate data? | |
| **Interpretations and Reminders** | |
| * Students will have to recall slope-intercept form and be able to interpret the slope and y-intercept in the context of the situation. * Students will need to be able to determine if a data set is linear or not in order to determine the rough line of best fit. | |
| **Misconceptions** | |
| * When creating scatterplots, students may believe that the 45-degree line always indicates a slope of 1. However, the variables may not have the same scaling, so they should be careful to calculate the slope based on two coordinate points, keeping scaling in mind.   [*http://www.katm.org/flipbooks/HS%20FlipBook%20Final%20CCSS%202014.pdf*](http://www.katm.org/flipbooks/HS%20FlipBook%20Final%20CCSS%202014.pdf) | |
| **Learning Progression (Suggested Learning Experiences)** | |